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Book Review: *Critical Properties of ϕ^4-Theories*

Critical Properties of ϕ^4-Theories. Hagen Kleinert, Verena Schulte-Frohlinde, World Scientific, Singapore, 2001.

Since the early 70's, the renormalization group (RG) approach has become one of the most reliable theoretical ways to perform analysis of the models of critical phenomena. In the last thirty years a number of valuable books have appeared on this subject, reflecting together with each author's views and original results, also different stages in the development of the theory. This recent book by Hagen Kleinert and Verena Schulte-Frohlinde reflects, to a considerable extent the current state-of-art in the RG theory of critical phenomena. Work in this theory may be extended by considering more realistic models to describe criticality and by proceeding to higher orders of perturbation theory to get reliable numerical results. More "realistic" models provide, on the microscopic level, an account of anisotropies of different types, of disorder, frustrations, etc. which finally lead to the consideration of models based on effective Hamiltonians of complicated symmetry. An account of higher orders of perturbation theory appears to be not solely a technical problem, since the resulting series have zero radius of convergence. Therefore appropriate resummation techniques need to be applied in order to get reliable data from these models. Both authors have made essential contributions to our common understanding of critical phenomena. Here, let me just mention a Kleinert criterion about onset of directional fluctuations in $O(N)$-symmetric theory which complements the Ginzburg criterion; a variational perturbational theory for critical exponents as an extension of Kleinert–Feynman variational approximation to path integrals; record expansions for the minimal subtraction RG functions for $O(N)$-symmetric theory and, later, for the theory with mixed $O(N)$-symmetric and cubic interactions; original techniques for resumming asymptotic series arising in the theory of critical phenomena. The list is far from being completed and the book provides an extension of this list and a natural summary of their work.

An impressive amount of ideas, formulas, and data is contained in 21 chapters of this book and in numerous appendices. At the beginning, the

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authors follow the traditional way in describing the field-theoretical RG technique in the theory of critical phenomena, introducing main physical quantities, describing the models, introducing the Feynman diagrams technique and explaining in details how to evaluate the diagrams (Chapters 1–6). Starting from the scalar ϕ^4 theory they pass then to the description of interactions with more complicated symmetries. However, the main part of the book is devoted to study of ϕ^4 theories with $O(N)$-symmetric and cubic interactions, a problem for which the authors have made major contributions. Scale transformation and renormalization are the subjects of Chapters 7–13. A definitely positive feature of the book is that the authors describe not only their own techniques (i.e., the minimal subtraction scheme) but rather give a general overview of existing alternatives for performing regularization and renormalization procedures.

The remainder of the book is devoted to explanation of calculation of the perturbation theory diagrams up to the order of five loops (Chapters 13–15) and to the introduction to the theory of resummation of asymptotic series with its application to the series arising in the theory of critical phenomena (Chapters 16–21). In particular, precise results are obtained for the critical exponents of the $O(N)$-symmetric theory and of the theory with combined $O(N)$ and cubic symmetries. Together with traditional techniques for resumming asymptotic series the authors describe original techniques successfully used for calculating critical exponents.

Inasmuch as the book contains both the backgrounds of the RG approach to the theory of critical phenomena as well as detailed technical recipes for dealing with high-order perturbation theory, the important task was to present the material in a manner most useful for the reader. Numerous appendices serve this purpose in the best possible way. They may also be considered as a separate sources for an interested reader. The same can be said about the reference lists. These are supplemented by detailed commentaries. They appear after each chapter, and the references give a clear picture of the state of the art in the subject.

In their preliminary remarks, the authors state that the purpose of the book is to introduce techniques for calculating power series for the critical exponents up to the order ε^5 and followed by resummation. This goal has certainly been achieved. This book is overall a very good one on the RG as applied to critical phenomena. I believe that it will soon achieve the status of a standard reference book on this subject.

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